A Minimal Arc-Geometric Reconstruction of Single-Electron Double-Slit Interference: From the $R = D_A$ Constraint to de Broglie Wavelength and Projection Density

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Truth need not be noisy; if it has force, it should be writable as definitions, theorems, and testable quantities.

Abstract

This manuscript develops a minimal mathematical reconstruction of single-electron double-slit interference within an arc-geometric framework. The model is not presented as a completed replacement for quantum mechanics. Rather, it asks a narrower and more testable question: can the core observable formulas of single-electron interference be recovered from a small set of geometric postulates involving an e-Arc-Helix, a zero-weight ring, and the steady coupling constraint $R = D_A$? We define D_A as an intrinsic arc-radius, not as an ordinary Euclidean diameter, and formulate arc phase closure, action-phase coupling, split-arc operators, branch coherence kernels, arc-difference fields, projection density, and measurement localization. Under these assumptions, the de Broglie relation $\lambda_A = h/p$, the two-slit bright-fringe condition $d \sin\theta = m\lambda_A$, and the continuous interference density $\rho_{\mathrm{arc}}(x) = |\mathcal{A}(x)|^2$ follow as conditional theorems. The manuscript then introduces a quantitative topology-friction functional, expressed through curvature mismatch, torsion mismatch, and $R - D_A$ coupling mismatch, and uses it to model self-decoherence and visibility decay. Finally, wave-function "collapse" is reinterpreted as the limiting case of environmental record formation and coherence-kernel suppression, $\rho_{\alpha\beta} \mapsto \mu_{\alpha\beta}\rho_{\alpha\beta}$, rather than as a primitive mysterious event. The goal is twofold: to show that Arc Theory can be disciplined into the conventional definition-axiom-theorem form, and to indicate a possible paradigm in which quantum probabilities are not primitive ontology but observable projection densities of a deeper geometric process.

Keywords: Arc Theory; e-Arc-Helix; single-electron interference; de Broglie wavelength; Born rule; projection density; topology friction; decoherence; measurement localization.

Contents

1 Introduction: a modest reconstruction problem

The double-slit experiment occupies a privileged position in the foundations of quantum mechanics. It joins a simple laboratory geometry to deep questions concerning interference, individuality, measurement, and the meaning of probability. The standard formalism predicts the observed distribution through complex amplitudes and the Born rule. In this manuscript we do not dispute the empirical success of that formalism. We ask a more constrained question:

> Can one formulate a minimal arc-geometric model which reproduces the standard single-electron double-slit observables while interpreting the Born density as a projection density rather than as primitive probability?

The background motivation comes from two prior Arc Theory notes. The first note proposed that single-electron self-interference, diffraction, and measurement localization can be understood as geometric projection phenomena involving a light-magnetic bipolar split, the $R = D$ constraint, the trace of ergodicity, self-decoherence, and a four-part operator structure. The second note supplied a short derivational chain from $R = D$ to the de Broglie relation and the two-slit condition. The present manuscript keeps the conceptual core but corrects and tightens the mathematical language. In particular, the symbol D is replaced by D_A, the intrinsic arc-radius or topological arc-radius, to avoid the ambiguity produced by calling D a diameter while writing a circumference-like expression $2\pi D$.

The paper is therefore not a manifesto against quantum mechanics. It is a conditional mathematical reconstruction. If the postulates stated below are admitted, then the standard formulas follow. If the postulates fail, the reconstruction fails. This is the proper scientific status of the present text.

The main contributions are:

(i) a cleaned definition of the $R = D_A$ steady coupling constraint;

(ii) a derivation of $\lambda_A = h/p$ from phase closure and action-phase coupling;

(iii) a split-arc reconstruction of the two-slit intensity law;

(iv) a phase-resolved projection-density interpretation of the Born rule;

(v) a quantitative topology-friction functional for self-decoherence;

(vi) a demystification of measurement collapse as coherence-kernel suppression by environmental records.

The structure deliberately follows the conventional mathematical-physics pattern: definitions, axioms, propositions, proofs, and limitations. This discipline is important. A new paradigm earns attention not by rejecting the old language, but by becoming clear enough to be tested in that language.

2 Conceptual background and methodological stance

2.1 What is preserved from the standard theory

The present model preserves the following standard empirical relations at the level considered here:

$$\lambda = \frac{h}{p}, \tag{1}$$

$$d \sin \theta = m\lambda, \qquad m \in \mathbb{Z}, \tag{2}$$

$$\rho(x) = |\psi(x)|^2. \tag{3}$$

Equation (1) is the de Broglie matter-wave relation. Equation (2) is the usual bright-fringe condition for a two-slit geometry. Equation (3) is the Born density, interpreted in ordinary quantum mechanics as a probability density. The present paper does not change these formulas in the minimal regime. Its novelty lies in their proposed geometric origin.

2.2 What is reinterpreted

The reinterpretation concerns ontology, not first-order experimental predictions in the two-slit limit. In the arc-geometric reading:

(a) the electron is not treated as a point corpuscle moving along a single classical path;

(b) nor is the probability wave taken as the primitive ontology;

(c) rather, the electron-like object is represented as an e-Arc-Helix whose phase-bearing trajectory is projected into a lower-dimensional measurement interface;

(d) the observed density is the phase-sensitive projection density of that arc process.

This does not eliminate statistics. It relocates statistics. Repeated localized detections remain statistical at the experimental interface, but the model interprets the density being sampled as a projection of an underlying geometric process.

2.3 Conditional rigor

All theorems below should be read as conditional. For example, the statement "$R = D_A$ implies $\lambda_A = h/p$" does not mean that $R = D_A$ has already been derived from a complete theory of matter, mass, charge, spin, and field coupling. It means that, within the minimal arc axioms, the de Broglie relation follows. This distinction is essential for intellectual honesty and for scientific usefulness.

3 Minimal arc-geometric setting

3.1 Arc manifold, arc trajectory, and projection

Definition 3.1 (Arc manifold and e-Arc-Helix)**.** *Let $\mathcal{M}_A$ be an arc manifold carrying a smooth measure $\mathrm{d}\mu_A$. An e-Arc-Helix is represented by a phase-bearing curve*

$$\Gamma_A : I \subset \mathbb{R} \to \mathcal{M}_A, \tag{4}$$

with local coordinate $q = \Gamma_A(\tau)$, intrinsic arc-density $\varrho_A(q) \geq 0$, and arc phase $\phi_A(q) \in \mathbb{R}/2\pi\mathbb{Z}$.

Definition 3.2 (Projection map)**.** *Let $\Pi : \mathcal{M}_A \to X$ be a projection map from the arc manifold to the detector configuration space X, usually an interval or plane on the screen. The measured coordinate is*

$$x = \Pi(q). \tag{5}$$

The Jacobian or local projection weight is denoted by $J_\Pi(q) \geq 0$.

The projection map formalizes the central idea that a high-dimensional or non-flat arc process is not observed directly. It is observed through a lower-dimensional measurement interface.

3.2 The meaning of D_A

Definition 3.3 (Intrinsic arc-radius)**.** *The symbol D_A denotes the intrinsic arc-radius or topological arc-radius of the e-Arc-Helix. It is a radius-like scale on the arc manifold, not a Euclidean diameter. The corresponding closed fundamental arc length is*

$$L_A = 2\pi D_A. \tag{6}$$

Remark 3.1 (Why the notation is changed)**.** *If D were an ordinary Euclidean diameter, a circumference would be πD, not $2\pi D$. The notation D_A avoids this error by declaring that the quantity entering $2\pi D_A$ is a radius-like arc scale. The original verbal intuition is retained, but the mathematical identity is cleaned.*

3.3 Zero-weight ring and steady coupling

Definition 3.4 (Zero-weight ring)**.** *The zero-weight ring is an idealized environmental curvature shell with radius $R > 0$ and closed length*

$$L_R = 2\pi R. \tag{7}$$

It is not assumed to be an ordinary material ring. It is the effective boundary scale through which the e-Arc-Helix is embedded into the measurement geometry.

Axiom 3.1 (Steady arc coupling)**.** *The minimal steady coupling condition is*

$$R = D_A. \tag{8}$$

This condition states equality between an external curvature radius and an intrinsic arc-radius. It does not assert that the radius and diameter of a single Euclidean circle are equal.

3.4 Arc phase and action

Axiom 3.2 (Phase closure)**.** *For a closed arc loop Γ, the arc phase satisfies*

$$\oint_\Gamma \mathrm{d}\phi_A = 2\pi n, \qquad n \in \mathbb{Z}. \tag{9}$$

The integer n is the winding number or phase-turn number of the closed arc loop.

Axiom 3.3 (Action-phase coupling)**.** *There exists an arc action S_A such that*

$$\mathrm{d}S_A = \hbar\, \mathrm{d}\phi_A. \tag{10}$$

The tangent momentum is defined by

$$p_A = \frac{\mathrm{d}S_A}{\mathrm{d}l}, \tag{11}$$

where l is intrinsic arc length along the relevant loop.

Remark 3.2 (Status of $\hbar$)**.** *In this minimal reconstruction, $\hbar$ remains a coupling constant between phase and action. The present work does not derive the numerical value of $\hbar$ from deeper arc dynamics. That task belongs to a future, stronger theory.*

3.5 Intrinsic phase pitch

Definition 3.5 (Arc wavelength or phase pitch)**.** *If the closed arc length is $L_A = 2\pi D_A$ and the phase turns n times around the loop, the intrinsic arc wavelength is the length per 2π phase turn:*

$$\lambda_A := \frac{L_A}{n} = \frac{2\pi D_A}{n}. \tag{12}$$

For the fundamental winding $n = 1$, this becomes $\lambda_A = 2\pi D_A$.

This definition is stronger than the informal statement $\lambda = 2\pi D$, because it also handles higher winding numbers and avoids the radius/diameter ambiguity.

4 From $R = D_A$ to de Broglie wavelength

4.1 Loop action quantization from phase closure

Lemma 4.1 (Loop action)**.** *Under phase closure (9) and action-phase coupling (10),*

$$\oint_\Gamma \mathrm{d}S_A = nh. \tag{13}$$

Proof. Using $\mathrm{d}S_A = \hbar \mathrm{d}\phi_A$,

$$\oint_\Gamma \mathrm{d}S_A = \hbar \oint_\Gamma \mathrm{d}\phi_A = \hbar(2\pi n) = nh. \tag{14}$$

□

Lemma 4.2 (Uniform tangent momentum)**.** *If p_A is constant along the steady loop of length $L_R = 2\pi R$, then*

$$p_A(2\pi R) = nh. \tag{15}$$

Proof. By definition, $\mathrm{d}S_A = p_A \mathrm{d}l$. Hence

$$\oint_\Gamma \mathrm{d}S_A = \oint_\Gamma p_A \mathrm{d}l = p_A \oint_\Gamma \mathrm{d}l = p_A(2\pi R). \tag{16}$$

Together with (13), this yields (15). □

4.2 The de Broglie relation as conditional theorem

Theorem 4.1 (Arc-geometric de Broglie relation)**.** *Assume phase closure, action-phase coupling, uniform tangent momentum on the steady loop, and the steady coupling $R = D_A$. Then the intrinsic arc wavelength satisfies*

$$\lambda_A = \frac{h}{p_A}. \tag{17}$$

Proof. From (15),

$$\frac{2\pi R}{n} = \frac{h}{p_A}. \tag{18}$$

By the steady coupling $R = D_A$,

$$\frac{2\pi D_A}{n} = \frac{h}{p_A}. \tag{19}$$

But $\lambda_A = 2\pi D_A/n$ by definition. Therefore

$$\lambda_A = \frac{h}{p_A}. \tag{20}$$

□

Corollary 4.1 (Fundamental winding)**.** *For $n = 1$,*

$$p_A(2\pi R) = h, \qquad \lambda_A = 2\pi D_A = 2\pi R = \frac{h}{p_A}. \tag{21}$$

Remark 4.1 (What has and has not been proved)**.** *The theorem shows that the de Broglie formula follows from the arc postulates. It does not by itself prove the postulates. In particular, it does not yet derive electron mass, charge, spin, magnetic moment, or the numerical value of $\hbar$. It establishes a conditional bridge from arc geometry to the standard matter-wavelength relation.*

5 Split-arc reconstruction and collapse demystified I

5.1 Boundary-induced split-arc operators

The two-slit apparatus is represented as a boundary condition rather than as a classical fork at which the electron must choose one slit. Let $\chi_1, \chi_2 : X \to [0, 1]$ be aperture functions localized at the two slits.

Definition 5.1 (Split-arc operator)**.** *The split-arc operator S_α, $\alpha = 1, 2$, acts on an arc state by*

$$(S_\alpha \Psi_A)(q) = \chi_\alpha(\Pi(q))\Psi_A(q). \tag{22}$$

The post-slit arc state is represented by

$$\Psi_A \longmapsto S_1\Psi_A + S_2\Psi_A. \tag{23}$$

Remark 5.1 (No particle fission)**.** *Equation (23) should not be read as a literal splitting of a point particle into two pieces. It states that a single arc process acquires two boundary-conditioned projection branches.*

5.2 Branch amplitudes and coherence kernel

At a detector point x, define branch amplitudes

$$\mathcal{A}_\alpha(x) = a_\alpha(x)\mathrm{e}^{\mathrm{i}\phi_\alpha(x)}, \qquad a_\alpha(x) \geq 0. \tag{24}$$

The coherent projection amplitude is

$$\mathcal{A}(x) = \mathcal{A}_1(x) + \mathcal{A}_2(x). \tag{25}$$

Definition 5.2 (Coherence kernel)**.** *The branch coherence kernel is*

$$\sigma_{\alpha\beta}(x) := \mathcal{A}_\alpha(x)\mathcal{A}_\beta(x)^*. \tag{26}$$

For two branches,

$$\sigma_{12}(x) = a_1(x)a_2(x)\mathrm{e}^{\mathrm{i}(\phi_1(x)-\phi_2(x))}. \tag{27}$$

5.3 Arc-difference field

Definition 5.3 (Arc-difference field)**.** *The arc-difference field between branches α and β is*

$$\Delta_{\alpha\beta}(x) := \phi_\alpha(x) - \phi_\beta(x). \tag{28}$$

For quasi-monochromatic propagation with arc wavelength λ_A,

$$\Delta_{12}(x) = \frac{2\pi}{\lambda_A}\Delta L(x), \tag{29}$$

where $\Delta L(x) = L_1(x) - L_2(x)$ is the geometric path difference.

5.4 Continuous two-slit intensity

Proposition 5.1 (Two-slit arc density)**.** *With the branch amplitudes (24), the phase-sensitive projection density is*

$$\rho_{\text{arc}}(x) = |\mathcal{A}(x)|^2 = a_1(x)^2 + a_2(x)^2 + 2a_1(x)a_2(x)\cos\Delta_{12}(x). \tag{30}$$

Proof. By (25),

$$|\mathcal{A}_1 + \mathcal{A}_2|^2 = (\mathcal{A}_1 + \mathcal{A}_2)(\mathcal{A}_1^* + \mathcal{A}_2^*) \tag{31}$$

$$= |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2 + \mathcal{A}_1\mathcal{A}_2^* + \mathcal{A}_1^*\mathcal{A}_2 \tag{32}$$

$$= a_1^2 + a_2^2 + 2a_1a_2\cos(\phi_1 - \phi_2). \tag{33}$$

□

This proposition is important because it replaces a too-strong statement sometimes made in informal arc language: that non-integer phase mismatch is simply "invisible." The mathematically correct statement is continuous: integer phase closure gives constructive maxima, half-integer mismatch gives destructive minima, and intermediate mismatch gives intermediate density.

5.5 Bright-fringe theorem

For the standard far-field two-slit geometry with slit separation d and observation angle θ,

$$\Delta L = d\sin\theta. \tag{34}$$

Theorem 5.1 (Two-slit bright-fringe condition)**.** *Under the arc-geometric de Broglie relation and the phase condition for constructive projection,*

$$\Delta_{12} = 2\pi m, \qquad m \in \mathbb{Z}, \tag{35}$$

bright fringes occur at

$$d\sin\theta = m\lambda_A. \tag{36}$$

Proof. Using (29) and (34),

$$\Delta_{12} = \frac{2\pi}{\lambda_A} d\sin\theta. \tag{37}$$

Constructive projection requires $\Delta_{12} = 2\pi m$. Hence

$$\frac{2\pi}{\lambda_A} d\sin\theta = 2\pi m, \tag{38}$$

which gives (36). □

Corollary 5.1 (Small-angle screen coordinate)**.** *If the screen is a distance L_s from the slits and $|\theta| \ll 1$, then*

$$y_m \simeq m\frac{\lambda_A L_s}{d} = m\frac{hL_s}{p_A d}. \tag{39}$$

5.6 First demystification of collapse

At this level, no mysterious collapse has been introduced. The detector intensity is determined by coherent branch addition. A localized detection event is not yet explained as a deterministic coordinate choice; rather, the model has explained the density sampled by many events. The language of collapse becomes necessary only when one asks why coherent branch information disappears under measurement. That issue is addressed in section 6 and section 7.

6 Topology friction: precise quantification of self-decoherence

This section is the central technical addition to the minimal reconstruction. The previous sections recover the de Broglie and double-slit formulas. The present section quantifies the arc-theoretic idea that forced projection into a flat measurement interface produces phase slip, geometric mismatch, and hence self-decoherence. The quantity modeling this process is called topology friction.

6.1 Mismatch fields

Let the e-Arc-Helix carry two conjugate internal components, denoted symbolically by $+$ and $-$, corresponding to the light-magnetic bipolar language of the original arc notes. Let

$$\kappa_+(l),\ \kappa_-(l) \tag{40}$$

be their projected curvature fields, and

$$\tau_+(l),\ \tau_-(l) \tag{41}$$

be their projected torsion fields along intrinsic length l. Define curvature and torsion mismatch by

$$\Delta\kappa_A(l) := \kappa_+(l) - \kappa_-(l), \tag{42}$$

$$\Delta\tau_A(l) := \tau_+(l) - \tau_-(l). \tag{43}$$

Define the dimensionless steady-coupling mismatch by

$$\varepsilon_R := \frac{R - D_A}{R}. \tag{44}$$

The three quantities $\Delta\kappa_A$, $\Delta\tau_A$, and ε_R represent three ways in which an ideal arc state can fail to remain perfectly self-conjugate under projection: curvature slip, torsion slip, and radius-coupling slip.

6.2 Topology-friction rate functional

Definition 6.1 (Topology-friction rate)**.** *Let $\gamma_\kappa, \gamma_\tau, \gamma_R > 0$ be phenomenological transport coefficients with dimensions chosen so that the final expression has units of inverse time. The internal topology-friction rate is defined by*

$$\Gamma_{\text{self}}[\Gamma_A, \Pi] := \int_{\Gamma_A} \left[\gamma_\kappa \big(\Delta\kappa_A(l)\big)^2 + \gamma_\tau \big(\Delta\tau_A(l)\big)^2 + \gamma_R \big(\varepsilon_R\big)^2\right] \mathrm{d}\mu_A(l). \tag{45}$$

This is the promised precise quantification. It states that self-decoherence is not a mystical loss of wave character. It is a nonnegative functional of geometric mismatch.

Proposition 6.1 (Non-negativity and zero-friction condition)**.** *For positive coefficients $\gamma_\kappa, \gamma_\tau, \gamma_R$,*

$$\Gamma_{\text{self}} \geq 0. \tag{46}$$

Moreover, $\Gamma_{\text{self}} = 0$ *if and only if, almost everywhere on the arc support,*

$$\Delta\kappa_A = 0, \qquad \Delta\tau_A = 0, \qquad R = D_A. \tag{47}$$

Proof. Each term in the integrand of (45) is a positive coefficient multiplied by a square. The integral of a nonnegative function is nonnegative. It vanishes exactly when each squared mismatch term vanishes almost everywhere on the support of $\mathrm{d}\mu_A$, giving (47). □

6.3 Steady coupling as a stable normal form

The same idea can be stated as a local energy or Lyapunov functional. Near a steady arc configuration, write the quadratic normal form

$$\mathcal{V}_A(R, D_A, \Delta\kappa_A, \Delta\tau_A) = \frac{K_R}{2}(R - D_A)^2 + \frac{K_\kappa}{2}\int_{\Gamma_A}(\Delta\kappa_A)^2 \mathrm{d}\mu_A + \frac{K_\tau}{2}\int_{\Gamma_A}(\Delta\tau_A)^2 \mathrm{d}\mu_A, \tag{48}$$

with $K_R, K_\kappa, K_\tau > 0$.

Theorem 6.1 (Local stability of the $R = D_A$ arc)**.** *The functional* (48) *has a unique local minimum at*

$$R = D_A, \qquad \Delta\kappa_A = 0, \qquad \Delta\tau_A = 0. \tag{49}$$

Proof. All terms in (48) are nonnegative. They vanish simultaneously only at (49). The second variation is positive in the directions controlled by the positive coefficients K_R, K_κ, K_τ, hence the point is a strict local minimum in this quadratic normal form. □

This theorem is modest but important. It does not derive the entire global dynamics of Arc Theory. It shows that the statement "$R = D_A$ is the zero-friction steady condition" has a mathematically disciplined expression.

6.4 Coherence-kernel decay law

Let $\sigma_{\alpha\beta}$ be the branch coherence kernel. Topology friction is postulated to suppress off-diagonal coherence according to

$$\frac{\mathrm{d}}{\mathrm{d}t}\sigma_{\alpha\beta}(t) = -\left[\Gamma^{\alpha\beta}_{\text{self}}(t) + \Gamma^{\alpha\beta}_{\text{ext}}(t) + \mathrm{i}\Omega^{\alpha\beta}_{\text{slip}}(t)\right]\sigma_{\alpha\beta}(t). \tag{50}$$

Here Γ_{ext} denotes ordinary external environmental decoherence, while Ω_{slip} is a reversible phase-slip frequency.

Solving (50) yields

$$\sigma_{\alpha\beta}(t) = \sigma_{\alpha\beta}(0)\exp\left[-\int_0^t(\Gamma^{\alpha\beta}_{\text{self}}(s) + \Gamma^{\alpha\beta}_{\text{ext}}(s))\mathrm{d}s\right]\exp\left[-\mathrm{i}\int_0^t \Omega^{\alpha\beta}_{\text{slip}}(s)\mathrm{d}s\right]. \tag{51}$$

Thus topology friction has a direct observable consequence: it reduces fringe visibility.

6.5 Visibility decay

For two branches with equal intensities $a_1 = a_2$, the initial visibility is one in the ideal coherent limit. Under topology friction and environmental decoherence,

$$V(t) = V(0) \exp\left[-\int_0^t (\Gamma^{12}_{\text{self}}(s) + \Gamma^{12}_{\text{ext}}(s))\mathrm{d}s\right]. \tag{52}$$

In the unequal-intensity case,

$$V(t) = \frac{2a_1a_2}{a_1^2 + a_2^2} \exp\left[-\int_0^t (\Gamma^{12}_{\text{self}}(s) + \Gamma^{12}_{\text{ext}}(s))\mathrm{d}s\right]. \tag{53}$$

Equations (45)–(53) are the precise form of the qualitative statement "topological friction causes self-decoherence." They also provide a path to empirical constraint: if isolated interferometry shows no residual intrinsic visibility loss beyond environmental decoherence, then the coefficients in (45) must be correspondingly small or the mismatch fields must vanish in that regime.

7 Measurement localization: collapse demystified II

7.1 Measurement as environmental record

The word "collapse" often suggests a discontinuous and unexplained event. In the present reconstruction, collapse is not primitive. It is the limiting case of environmental record formation and off-diagonal coherence suppression.

Let $|\Gamma_1\rangle, |\Gamma_2\rangle$ be two branch states and $|E_1\rangle, |E_2\rangle$ the corresponding environmental record states. Before environmental tracing, a branch-entangled state may be written as

$$|\Psi_{AE}\rangle = c_1|\Gamma_1\rangle|E_1\rangle + c_2|\Gamma_2\rangle|E_2\rangle. \tag{54}$$

Tracing out the environment yields reduced branch coherences proportional to

$$\langle E_2|E_1\rangle. \tag{55}$$

If the environment stores no which-branch record, $|E_1\rangle$ and $|E_2\rangle$ are nearly identical and the overlap is near one. If the environment stores a robust record, the overlap is near zero.

7.2 Measurement localization operator

Definition 7.1 (Measurement localization operator)**.** *The measurement localization operator Λ_D acts on the branch density matrix by*

$$\Lambda_D : \rho_{\alpha\beta} \longmapsto \mu_{\alpha\beta}\rho_{\alpha\beta}, \tag{56}$$

where

$$\mu_{\alpha\beta}(t) := \langle E_\beta(t)|E_\alpha(t)\rangle \exp\left[-\int_0^t \Gamma^{\alpha\beta}_{\text{self}}(s)\mathrm{d}s\right]. \tag{57}$$

The complex number $\mu_{\alpha\beta}$ is the total coherence survival factor. Its magnitude controls visibility; its phase shifts fringes.

7.3 Measured two-slit density

Including measurement localization, the two-slit density becomes

$$\rho_{\text{arc}}(x) = a_1^2 + a_2^2 + 2a_1a_2 \,\text{Re}\left[\mu_{12}\mathrm{e}^{\mathrm{i}\Delta_{12}(x)}\right]. \tag{58}$$

Equivalently,

$$\rho_{\text{arc}}(x) = a_1^2 + a_2^2 + 2a_1a_2|\mu_{12}|\cos\big(\Delta_{12}(x) + \arg\mu_{12}\big). \tag{59}$$

Theorem 7.1 (Collapse as the zero-coherence limit)**.** *If $|\mu_{12}| \to 0$, then the interference term in (59) vanishes and*

$$\rho_{\text{arc}}(x) \longrightarrow a_1(x)^2 + a_2(x)^2. \tag{60}$$

Thus the disappearance of interference is the limiting case of coherence-kernel suppression.

Proof. The interference contribution in (59) is proportional to $|\mu_{12}|$. Taking $|\mu_{12}| \to 0$ removes this term, leaving (60). □

This is the promised demystification. Collapse is not introduced as an additional metaphysical operation. It is represented as the limiting behavior of a measurable coherence factor.

7.4 Visibility and distinguishability

The fringe visibility associated with (59) is

$$V = \frac{\rho_{\max} - \rho_{\min}}{\rho_{\max} + \rho_{\min}} = \frac{2a_1a_2|\mu_{12}|}{a_1^2 + a_2^2}. \tag{61}$$

For equal branch weights, $a_1 = a_2$,

$$V = |\mu_{12}|. \tag{62}$$

A compatible path distinguishability parameter is

$$D_{\text{path}} = \sqrt{1 - |\mu_{12}|^2}, \tag{63}$$

so that for equal branch weights

$$V^2 + D_{\text{path}}^2 = 1. \tag{64}$$

In nonideal cases, loss, imbalance, and additional noise weaken this equality to the familiar inequality form.

7.5 Physical reading

Equations (56)–(64) give a compact arc-geometric account of measurement:

(i) the branch structure is created by boundary-conditioned projection, not by literal particle duplication;

(ii) the coherence kernel carries the possibility of interference;

(iii) topology friction and environmental records suppress the off-diagonal kernel;

(iv) localized detections are the detector-interface events drawn from the remaining projection density;

(v) the apparent collapse is the operational face of $\mu_{\alpha\beta} \to 0$.

No appeal to consciousness or observer mysticism is required.

8 Projection density and the Born rule

8.1 Raw occupancy is not enough

A purely classical-looking trajectory occupancy would be

$$\rho_{\mathrm{occ}}(x) = \lim_{T\to\infty} \frac{1}{T} \int_0^T \delta\big(x - \Pi(\Gamma_A(\tau))\big)\mathrm{d}\tau. \tag{65}$$

This expression captures the "trace of ergodicity" intuition, but by itself it is not sufficient for interference, because interference is phase-sensitive. The full arc-geometric density must retain phase before taking the observable modulus.

8.2 Phase-resolved projection amplitude

Definition 8.1 (Phase-resolved projection amplitude)**.** *Let δ_ϵ be a mollified delta function on the detector space, with ϵ the resolution scale of the measurement interface. The phase-resolved projection amplitude is*

$$\mathcal{A}_\epsilon(x) := \int_{\Gamma_A} \sqrt{\varrho_A(q)}\, \mathrm{e}^{\mathrm{i}\phi_A(q)} \delta_\epsilon\big(x - \Pi(q)\big) J_\Pi(q)^{1/2}\, \mathrm{d}\mu_A(q). \tag{66}$$

The idealized amplitude is the distributional limit

$$\mathcal{A}(x) = \lim_{\epsilon\to 0} \mathcal{A}_\epsilon(x), \tag{67}$$

when the limit is meaningful in the weak sense.

Definition 8.2 (Arc projection density)**.** *The observable arc projection density is*

$$\rho_{\mathrm{arc}}(x) := |\mathcal{A}(x)|^2. \tag{68}$$

8.3 Born density as projection density

Proposition 8.1 (Born form)**.** *If the standard wavefunction $\psi(x)$ is identified with the normalized phase-resolved projection amplitude $\mathcal{A}(x)$, then*

$$|\psi(x)|^2 = \rho_{\mathrm{arc}}(x). \tag{69}$$

Proof. The statement follows directly from the identification $\psi = \mathcal{A}/\|\mathcal{A}\|$ and the definition (68). The nontrivial content lies in the construction of $\mathcal{A}$ as a phase-resolved projection of an arc process rather than as a primitive probability amplitude. □

Remark 8.1 (Interpretive status)**.** *Equation* (69) *is not a denial of experimental probability. It is a proposed reinterpretation of the density that experiments sample. In ordinary language, $|\psi|^2$ is the probability density for detection. In the present model, the same object is the normalized projection density of an underlying phase-bearing arc process.*

9 The four-part operator skeleton

The minimal reconstruction can be summarized by a four-part operator skeleton:

$$\boxed{\mathcal{M}_A = \Lambda_D \circ \Delta_{\alpha\beta} \circ \sigma_{\alpha\beta} \circ S_\alpha.} \tag{70}$$

The order in (70) is conceptual rather than a literal composition of operators on the same space. Each part has a defined role:

Component	**Mathematical role**	**Physical interpretation**
S_α	Boundary-conditioned branch projection $(S_\alpha\Psi)(q) = \chi_\alpha(\Pi(q))\Psi(q)$	A slit does not split a particle; it splits the allowed projection branches of one arc state.
$\sigma_{\alpha\beta}$	Coherence kernel $\mathcal{A}_\alpha\mathcal{A}_\beta^*$	Off-diagonal branch coherence carries interference capacity.
$\Delta_{\alpha\beta}$	Phase or arc-difference field $\phi_\alpha - \phi_\beta$	Geometric mismatch determines constructive and destructive projection.
Λ_D	Localization/decoherence map $\rho_{\alpha\beta} \mapsto \mu_{\alpha\beta}\rho_{\alpha\beta}$	Measurement is environmental record formation and local locking, not metaphysical collapse.

This table transforms the original four-part intuition into a minimal technical architecture. A future complete theory would need to specify the full function spaces, domains, codomains, and dynamical equations for these objects.

10 Comparison with standard quantum mechanics

10.1 Empirical equivalence in the minimal two-slit regime

In the regime covered by this paper, the predictions coincide with standard quantum mechanics:

$$\lambda_A = h/p_A, \tag{71}$$

$$d\sin\theta = m\lambda_A, \tag{72}$$

$$\rho_{\mathrm{arc}}(x) = |\mathcal{A}_1(x) + \mathcal{A}_2(x)|^2, \tag{73}$$

$$\rho_{\mathrm{arc}}(x) = a_1^2 + a_2^2 + 2a_1a_2|\mu_{12}|\cos(\Delta_{12} + \arg\mu_{12}). \tag{74}$$

Therefore, the minimal model should not be advertised as an immediate empirical overthrow of standard quantum mechanics. Its claim is subtler: it offers a geometric ontology that reproduces the standard formulas while reassigning the meaning of $|\psi|^2$.

10.2 Where the interpretation differs

The standard operational statement is:

$$|\psi(x)|^2 = \text{probability density of detecting the particle at } x. \tag{75}$$

The arc-geometric statement is:

$$|\mathcal{A}(x)|^2 = \text{projection density of a phase-bearing arc process at } x. \tag{76}$$

The same detection statistics may arise, but the ontology is different. In this sense, Arc Theory in the present form is not a rival formula for the two-slit pattern; it is a rival interpretation of the source of the formula.

10.3 Why topology friction is more than a metaphor

The phrase "topology friction" would be scientifically weak if it remained only qualitative. In section 6 it was turned into a nonnegative functional whose terms have explicit geometric meaning. The claim can now be constrained by experiment. In an ideally isolated apparatus, if no intrinsic loss of visibility is observed, then

$$\int_0^t \Gamma_{\mathrm{self}}^{12}(s)\mathrm{d}s \approx 0 \tag{77}$$

over the relevant time interval. This means either the mismatch fields vanish in the experimental geometry or the coefficients $\gamma_\kappa, \gamma_\tau, \gamma_R$ are sufficiently small. A theory that can be bounded by experiment has entered the scientific conversation more seriously than a pure metaphor.

11 Falsifiability, scope, and limitations

11.1 Directly reproduced tests

The minimal reconstruction reproduces the following standard dependencies:

(a) changing electron momentum changes the wavelength through $\lambda_A = h/p_A$;

(b) changing slit separation d shifts fringe spacing according to $d \sin\theta = m\lambda_A$;

(c) changing screen distance L_s gives $y_m \simeq m\lambda_A L_s/d$;

(d) introducing which-path information reduces visibility through $|\mu_{12}|$.

11.2 Potentially distinctive tests

The distinctive content of the present model lies mainly in topology friction. Possible tests would seek residual, geometry-dependent loss of visibility not attributable to known environmental decoherence:

$$V(t) = V_{\mathrm{env}}(t) \exp\left[-\int_0^t \Gamma_{\mathrm{self}}^{12}(s)\mathrm{d}s\right]. \tag{78}$$

If apparatus modifications alter projected curvature or torsion mismatch while controlling ordinary decoherence, the model predicts corresponding changes in the intrinsic visibility factor. If no such dependence is found, the topology-friction coefficients are constrained or the proposed mismatch fields are physically irrelevant.

11.3 Limitations intentionally acknowledged

This manuscript does not derive:

(i) the numerical value of $\hbar$;

(ii) the electron rest mass;

(iii) the electron charge;

(iv) spin-$1/2$ and fermionic exchange statistics;

(v) the magnetic moment or the electron g-factor;

(vi) a relativistic field theory for the arc state;

(vii) a full many-body theory.

These are not minor details. They are the tests that any complete arc-geometric theory of matter must eventually face. The present manuscript should therefore be read as a minimal reconstruction of single-electron interference, not as a completed theory of all quantum phenomena.

12 Paradigmatic implications

The traditional paradigm treats the wavefunction as a mathematical object whose squared modulus gives probabilities. Many interpretations then debate what the wavefunction "really" is. The arc-geometric proposal changes the direction of inquiry: perhaps the wavefunction is not the final object requiring interpretation, but the projection shadow of a deeper phase-bearing geometry.

This does not require contempt for the traditional formalism. On the contrary, the formalism is used as a constraint. Any new paradigm worth considering must first pass through the old equations without breaking them. The present manuscript attempts exactly that for the simplest and most emblematic case: single-electron double-slit interference.

The philosophical shift may be summarized as follows:

Traditional operational reading	**Arc-geometric ontological reading**
$\lvert\psi\rvert^2$ is detection probability.	$\lvert\mathcal{A}\rvert^2$ is projection density sampled as probability.
Interference comes from amplitude superposition.	Interference comes from phase-bearing branch projection of one arc process.
Collapse is a postulate or interpretive problem.	Collapse is the limiting case $\mu_{\alpha\beta} \to 0$ of coherence-kernel suppression.
Decoherence is environmental loss of phase relation.	Decoherence includes environmental loss plus quantifiable topology friction.

The new paradigm is therefore not noise against the old. It is an invitation to ask whether the old probabilities can be read as shadows of a deeper geometry.

13 Conclusion

This paper has constructed a minimal arc-geometric reconstruction of single-electron double-slit interference. The central chain is

$$\oint \mathrm{d}\phi_A = 2\pi n, \qquad \mathrm{d}S_A = \hbar \mathrm{d}\phi_A, \qquad R = D_A \quad \Longrightarrow \quad \lambda_A = \frac{h}{p_A}, \tag{79}$$

followed by

$$\Delta_{12} = \frac{2\pi}{\lambda_A} d \sin\theta \quad \Longrightarrow \quad d\sin\theta = m\lambda_A. \tag{80}$$

The observed density is expressed as

$$\rho_{\mathrm{arc}}(x) = \lvert\mathcal{A}(x)\rvert^2, \tag{81}$$

where $\mathcal{A}(x)$ is a phase-resolved projection amplitude of the arc process. Measurement localization is represented by

$$\rho_{\alpha\beta} \mapsto \mu_{\alpha\beta}\rho_{\alpha\beta}, \tag{82}$$

with collapse appearing as the limit $|\mu_{\alpha\beta}| \to 0$. Topology friction is quantified as

$$\Gamma_{\text{self}} = \int_{\Gamma_A} \left[\gamma_\kappa (\Delta\kappa_A)^2 + \gamma_\tau (\Delta\tau_A)^2 + \gamma_R \left(\frac{R - D_A}{R} \right)^2 \right] \mathrm{d}\mu_A. \tag{83}$$

This is the most compact mathematical statement of the present reconstruction.

The paper has also emphasized its own limits. It does not yet derive the constants and particle properties needed for a complete theory. Its achievement is more specific: it shows that the arc-theoretic intuition can be translated into a conventional mathematical form that reproduces the standard two-slit relations while opening a coherent possibility for a non-probabilistic geometric ontology beneath the observed probability law.

A Notation table

Symbol	**Meaning**
$\mathcal{M}_A$	Arc manifold.
Γ_A	e-Arc-Helix trajectory or support curve.
Π	Projection map from arc manifold to detector space.
R	Zero-weight ring radius or external curvature scale.
D_A	Intrinsic arc-radius/topological arc-radius; not a Euclidean diameter.
ϕ_A	Arc phase.
S_A	Arc action.
p_A	Tangent momentum defined by $p_A = \mathrm{d}S_A/\mathrm{d}l$.
λ_A	Arc phase pitch/wavelength.
S_α	Split-arc operator induced by aperture boundary α.
$\mathcal{A}_\alpha$	Branch projection amplitude.
$\sigma_{\alpha\beta}$	Branch coherence kernel.
$\Delta_{\alpha\beta}$	Arc-difference phase field.
Λ_D	Measurement localization operator.
$\mu_{\alpha\beta}$	Coherence survival factor, including environmental overlap and self-decoherence.
Γ_{self}	Topology-friction self-decoherence rate.
Γ_{ext}	External environmental decoherence rate.

B Dimensional comment on topology friction

In (45), curvature and torsion have dimensions of inverse length. Thus $(\Delta\kappa_A)^2\mathrm{d}\mu_A$ and $(\Delta\tau_A)^2\mathrm{d}\mu_A$ have dimensions of inverse length if $\mathrm{d}\mu_A$ is an arc-length element. The coefficients γ_κ and γ_τ therefore carry dimensions of length per time. The term $\varepsilon_R^2\mathrm{d}\mu_A$ has dimensions of length, so γ_R carries dimensions of inverse length per time. These assignments ensure that Γ_{self} is a rate.

C Reference form of the minimal derivation

For convenience, the full minimal chain is collected here:

$$\oint_\Gamma \mathrm{d}\phi_A = 2\pi n, \tag{84}$$
$$\mathrm{d}S_A = \hbar \mathrm{d}\phi_A, \tag{85}$$
$$\oint_\Gamma \mathrm{d}S_A = nh, \tag{86}$$
$$\mathrm{d}S_A = p_A \mathrm{d}l, \tag{87}$$
$$p_A(2\pi R) = nh, \tag{88}$$
$$\lambda_A = \frac{2\pi D_A}{n}, \tag{89}$$
$$R = D_A, \tag{90}$$
$$\lambda_A = \frac{h}{p_A}, \tag{91}$$
$$\Delta_{12} = \frac{2\pi}{\lambda_A}\Delta L, \tag{92}$$
$$\Delta L = d\sin\theta, \tag{93}$$
$$\Delta_{12} = 2\pi m \Longrightarrow d\sin\theta = m\lambda_A, \tag{94}$$
$$\rho_{\mathrm{arc}}(x) = |\mathcal{A}_1(x) + \mathcal{A}_2(x)|^2, \tag{95}$$
$$\Lambda_D : \rho_{\alpha\beta} \mapsto \mu_{\alpha\beta}\rho_{\alpha\beta}. \tag{96}$$

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