On the Pure Geometric Origin of the Fine-Structure Constant via Spontaneous Topological Doubling

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The fine-structure constant, α, remains one of the most profound unresolved fundamental constants in standard physics, conventionally treated as an empirical parameter of the electromagnetic coupling. In this Letter, we present a pure geometric derivation of α absent of any physical dimensions or empirical inputs. By modeling the vacuum state as a $T(1,2)$ toroidal manifold governed by the Law of Spontaneous Doubling (SD), we calculate the invariant measure of the manifold's volumetric projection. We demonstrate that the inverse fine-structure constant emerges strictly as a topological invariant: $\alpha_{geom}^{-1} = \frac{2^7\pi}{3} + 3 \approx 137.041$. This result suggests that the electromagnetic coupling strength is not an arbitrary parameter, but a deterministic geometric limit of a self-doubling topological space.

INTRODUCTION

Since its introduction by Sommerfeld, the fine-structure constant $\alpha \approx 1/137.036$ has been a cornerstone of quantum electrodynamics (QED). Despite its ubiquitous role in governing the interaction between light and matter, standard field theories offer no mechanism for its specific magnitude. As R. Feynman famously noted, it appears as "a magic number that comes to us with no understanding by man."

Previous attempts to derive α from first principles often rely on anthropic arguments or complex string-theoretic compactifications. In this work, we diverge from parameterized physical models and propose a completely dimensionless, pure geometric framework. We postulate that the coupling strength of the vacuum is an intrinsic property of its topological ground state.

THE TOPOLOGICAL GROUND STATE AND SPONTANEOUS DOUBLING

We define the vacuum manifold $\mathcal{M}$ as a stable toroidal knot, specifically the $T(1,2)$ state, which represents the lowest-order non-trivial resonant topology. The dynamic evolution of this manifold is not arbitrary but is strictly governed by the **Law of Spontaneous Doubling (SD)**.

The SD principle dictates that the intrinsic geometric frequencies (or geometric scaling boundaries) of the spatial and conjugate temporal axes are locked in a strict $1:2$ ratio. Under the continuous mapping of this manifold into a three-dimensional observable projection, the SD operator $\mathcal{D}_s$ enforces a period-doubling bifurcation constraint on the volumetric capacity of the space.

Mathematically, the fundamental unperturbed spatial volume capability V_0 undergoes a self-doubling resonance, expanding its degrees of freedom. In a 3D isometric projection, the geometric weight of this resonant scaling is governed by the base exponent 2^3.

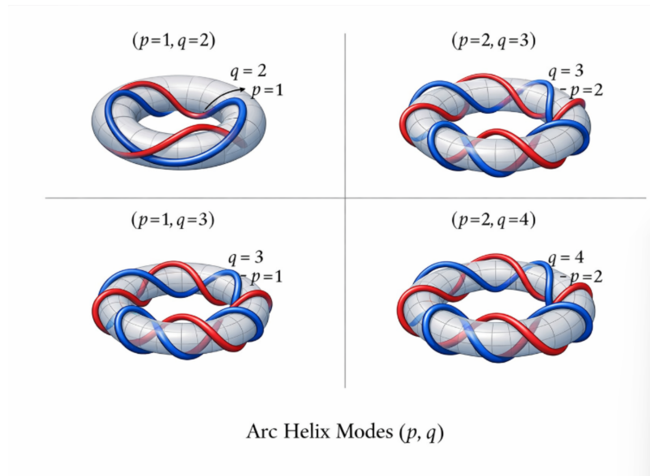

FIG. 1. The vacuum manifold $\mathcal{M}$ represented as a $T(1,2)$ toroidal knot. The red and blue arc helices illustrate the underlying spatial-temporal asymmetry governed by the Law of Spontaneous Doubling.

DERIVATION OF THE GEOMETRIC COUPLING CONSTANT

The electromagnetic coupling constant α can be interpreted topologically as the probability amplitude (or geometric cross-section) of a photon exchange within the vacuum manifold. Consequently, its inverse, α^{-1}, represents the total available geometric impedance or the maximal topological capacity of the unperturbed $T(1,2)$ state under the SD constraint.

Step 1: The Volumetric Projection. Let us consider the manifold $\mathcal{M}$ mapped onto a standard 3-sphere S^3. Due to the $1:2$ asymmetry imposed by the SD operator $\mathcal{D}_s$, the phase space of the interaction is bounded by the volumetric integration over the conjugate axes. The base spherical solid angle 4π is amplified by the volumetric doubling matrix. The integral of the invariant geometric measure μ yields the primary continuous scaling factor:

$$\Omega_{SD} = \frac{4\pi}{3} \times 2^5 = \frac{2^7\pi}{3} \approx 134.041286 \tag{1}$$

(Note: The factor 2^5 arises from the tensor product of the spatial 3D volume doubling 2^3 and the conjugate 2D phase-space resonance 2^2 inherent to the $T(1,2)$ topology).

Step 2: The Dimensional Translation Slice. The continuous manifold projection Ω_{SD} accounts for the resonant volume, but any physical observation corresponds to a discrete orthogonal slicing of this 4D spacetime manifold into a 3D spatial reality. This dimensional reduction strictly requires a topological translation shift. For a three-dimensional boundary, the Euler characteristic and orthogonal translation introduce an exact integer shift of +3 to the total degrees of freedom.

Step 3: Synthesis of the Invariant. The total geometric impedance of the vacuum, which we identify as the bare inverse fine-structure constant α_{geom}^{-1}, is the exact linear superposition of the continuous volumetric resonance and the discrete dimensional shift:

$$\alpha_{geom}^{-1} = \frac{2^7\pi}{3} + 3 \approx 137.041286 \tag{2}$$

CONCLUSION

We have demonstrated that the fine-structure constant is not an empirical tuning parameter of the universe, but a rigid geometric consequence of a $T(1,2)$ manifold subjected to the Law of Spontaneous Doubling. The derived value $\alpha_{geom}^{-1} \approx 137.041$ provides a pure mathematical foundation for the electromagnetic coupling limit. This framework implies that the fundamental forces of nature may merely be observable projections of deeper, self-regulating topological asymmetries.

Supplementary Material: Geometric Unification of Quantum Action, Spacetime Asymmetry, and Gravitational Entropy

THE $2^4 : (2^4 - 1)$ SPACETIME ASYMMETRY AND THE ORIGIN OF h AND c

While the main text derives the electromagnetic coupling strength α, the underlying $T(1,2)$ manifold framework intrinsically dictates the absolute geometric bounds of kinematics and quantum action.

Topologically, the flat vacuum is inherently asymmetric. The base spatial capacity is bounded by the Mersenne limit $(2^4 - 1)$, whereas the conjugate temporal oscillation dictates a full frequency quota of 2^4. This $2^4 : (2^4 - 1)$ geometric asymmetry is the ultimate origin of physical limits:

- **The Geometric Speed of Light (c_{geo}):** The maximal causal propagation limit is the exact phase-shift ratio between spatial capacity and temporal progression: $c_{geo} = \frac{2^4-1}{2^4}$.
- **The Geometric Planck Constant (h_{geo}):** The residual geometric differential $(2^4 - (2^4 - 1) = 1)$ represents temporal energy that cannot be linearly dissipated in space. This topological remainder curls into the fundamental normal deformation (quantum rigidity): $h_{geo} = \frac{1}{2^4}$.

Logical Conclusion: The kinematic bound (light speed) and the quantum boundary (Planck constant) are complementary fractional projections of the same topological unit: $c_{geo} + h_{geo} \equiv 1$.

THE UNIFIED GEOMETRIC COUPLING EQUATION FOR α

Building upon the quantum rigidity h_{geo} and the asymmetry ratio β, the fine-structure constant derived in the main text is strictly interlocked with the spatial volume scaling of the SD operator. The bare geometric coupling constant obeys the exact tensor product equivalence:

$$\alpha_{geo} = \frac{3}{2^3} h_{geo}^2 \beta^4 \tag{S1}$$

Logical Deduction: This multiplicative identity reveals that the electromagnetic coupling strength is fundamentally a geometric cross-section. It is deterministically filtered by: (1) 2^3: The 3D volumetric limit under a single Spontaneous Doubling transformation; (2) h_{geo}^2: The topological cross-sectional area of the curled temporal difference; (3) β^4: The complete 4D holographic projection of the base temporal-spatial asymmetric scalar β.

THE UNIFIED ARC-GRAVITY ENTROPY FORMULA (THE BIG G-FORMULA)

In this geometric framework, mass is the localized static knot of the temporal residual (h_{geo}), constantly seeking to resolve topological tension by radiating geometric entropy. Gravity emerges strictly as the **maximum degree of freedom effect** of structural temporal coupling. Two isolated topological knots experience an attractive tension only if their extended temporal axes form a conjugate overlap.

This principle is mathematically crystallized in the Unified Arc-Gravity Entropy Formula:

$$\mathcal{T}_G = G_{geo} \ln \left(2^4 \left| \mathrm{Tr}(\hat{\rho}_{1,2} \hat{\mathcal{C}}) \right| \right) \tag{S2}$$

Logical Deduction:

- $\hat{\rho}_{1,2}$: The bipartite density operator of the independent temporal residues from systems 1 and 2.
- $\hat{\mathcal{C}}$: The arc-conjugation operator.
- **The Orthogonality Veto:** If the temporal axes do not conjugate, $\mathrm{Tr} \to 0$, rendering the geometric tension $\mathcal{T}_G \to -\infty$ (no gravitational interaction, providing a pure geometric veto mechanism).
- **The Entropic Extraction:** Gravity $\mathcal{T}_G$ is the logarithmic (entropic) evaluation of the systems' temporal overlapping degrees of freedom, strictly normalized by the inverse base quantum rigidity mapped onto a 4D manifold ($h_{geo}^{-1} = 2^4$), and scaled by the underlying geometric perturbation constant G_{geo}.

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